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Note

Algorithms for Eulerian Treatment of Jet Breakup Induced by Surface Tension

INTRODUCTION

In a recent paper [1], to which the reader should refer for more details, we presented a numerical investigation of the disintegration of a liquid jet, subject to capillary forces, and emanating into a stationary, low-density ambient. The full Navier–Stokes equations were employed. As in the previous theoretical studies of other investigators, we used the idealization of an infinite jet (Lagrangian coordinates). In the present extension of our earlier work, that assumption is abandoned (Eulerian coordinates employed). The resulting computational problem is much more difficult, but more representative of many real-world applications.

The principal difficulties associated with the Eulerian approach lie, of course, in realistic simulation of the "entrance" and "exit" conditions for the computational field—particularly for the free surface. Those algorithms which we found to provide a successful simulation are described below. Most of these were runtime generated by the computer, using a new technique [1, 2].

The computations were performed on a cylindrical column of liquid emanating in the z-direction from a long-throat, circular nozzle of radius, $r = a$. The initial jet configuration and the velocities ($u = v_r$; $w = v_z$) were assumed, and the ensuing transient phenomena computed.

The computation scheme was based on the following axisymmetric, Stokes stream-function-vorticity (Ψ, ω) relations (Eqs. (1) and (2)) (ν, kinematic viscosity):

$$d(r\omega)/dt - 2u\omega = \nu\, D(r\omega) \tag{1}$$

$$D\Psi = r\omega, \tag{2}$$

where:

$$d/dt = \partial/\partial t + u\,\partial/\partial r + w\,\partial/\partial z \tag{3}$$

$$D = \partial^2/\partial r^2 - (1/r)\,\partial/\partial r + \partial^2/\partial z^2. \tag{4}$$

Since the vorticity consists of second-order spatial derivatives of the stream function, Eq. (1) was converted to finite-difference form in a very straightforward manner, with the use of central space differences and ADI operating on a simple forward time difference. On the other hand, considerable care had to be exercised in the generation of the algorithms for Ψ itself.

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Formulation of Algorithms

In the computations, the time, t, was scaled by $\rho a^3/\sigma$ (ρ, density; σ, surface tension), axial distance, z, was scaled by the nozzle radius, and transverse, radial distance, r, was scaled (coordinate straining) by the *local* jet radius, making $\eta = r/r_s(z, t)$. Such non-dimensionalization will hereafter be used.

The Ψ algorithms were formulated in terms of (η, z) for the nine-point grid shown in Fig. 1. These algorithms were not developed in the usual manner by substituting for derivatives their finite-difference expressions. Rather, the Stokesian stream function, Ψ, was approximated by a high-order spatial polynomial (in general, 17 terms), made to satisfy at each of the nine grid points either the (Ψ, ω) differential equation (1) or a boundary condition. Over-constraint of the system of locally-linear equations results in an equation of condition, namely, an algorithm involving the grid point values of Ψ, ω, etc. The foregoing operation, too tedious and complicated to be performed "by hand," is accomplished as an integral part of the overall computation process, with the coefficients of the algorithm being continually revised as functions of space and time. In addition to precision, stability near boundaries is obtained by the inherent close coupling of the boundary conditions with the differential equation. References [1, 2] must be consulted for details.

The present computational field is bounded by (a) an "entrance" at the nozzle exit, (b) an "exit" *sufficiently* far downstream, (c) the jet axis, and (d) the free surface. The general solution techniques for Eqs. (1) and (2), and the treatment of boundary conditions for the centerline and free surface used here were the same as in [1, 1]. Only the extrance, exit, and "corner" conditions differed, and these will now be discussed.

Entrance

The orifice of the nozzle was taken as the "entrance" section of the computational field, with stream function, velocities, and vorticity corresponding to a Poiseuille flow. With greater accuracy, but greater computer time, the entrance might have been taken upstream within the nozzle. Thus the values of Ψ and ω at points 1, 2, and 3 were held fixed, while the values of the other points of the grid were taken as they developed with time.

Exit

The computational field must necessarily be finite. Selection of appropriate outlet conditions proved to be the most challenging part of the entire problem. At outlet points 7, 8, and 9 the radial pressure derivative was set equal to zero and the vorticity was obtained by quadratic extrapolation from three upstream neighbors possessing the same value of η. The adopted pressure condition, typically realized in boundary layers, is also acceptable for drop formation. It imposes a condition on Ψ through the radial momentum equation.

At other points of the grid, the usual Ψ and ω specifications were made.

Free Surface-Entrance Corner

Let point 1 of Fig. 1 denote the "corner" where the free surface (1, 4, 7) attaches to the nozzle. At the wall, the shear stress has its Poiseuille value, whereas on the free surface the shear stress is negligible. Thus there is a discontinuity in this stress. We found, however, that the general Ψ-polynomial supported this variation satisfactorily when the following specifications were made.

At points 1, 2, and 3 constant values of Ψ and ω were specified. At points 4 and 7, the "standard" surface tension and zero shear stress conditions were set (see [1, 2]). Ψ and ω were taken at the interior points, varying, of course as the solution proceeded.

Free Surface-Exit Corner

Let points 1, 4, and 7 denote the surface and 7, 8, and 9 denote the exit. The cubic spline fit for $r_s(z, t)$ is terminated by requiring the third derivative to be continuous at point 4. The surface tension condition, and that of zero shear stress, are applied at points 1 and 4. At the corner point, zero shear stress and zero radial pressure gradient are applied. At points 8 and 9 the regular exit conditions are used, and standard Ψ and ω values are set at points 2, 3, 5.

Numerical Results

Calculations were made for a jet of water of 0.0035 cm in diameter, with a mean stream velocity of 50 cm/s, a Reynolds number of 17.5, and a surface tension of 72.5 dyne/cm. For initial conditions, a Poiseuille velocity distribution (used in Fig. 2) was generally assumed for the entire jet. Because of the precision of the finite-difference algorithms, relatively large spatial increments, $\Delta z = 0.4$ and $\Delta\eta = 0.1$, could be used. For the most part, a temporal increment of $\Delta t = 0.1$ was employed, although when jet "pinchoff" commenced, this increment was reduced.

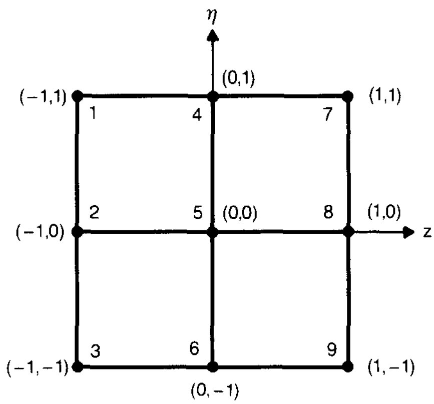

FIG. 1. Nine-point grid.

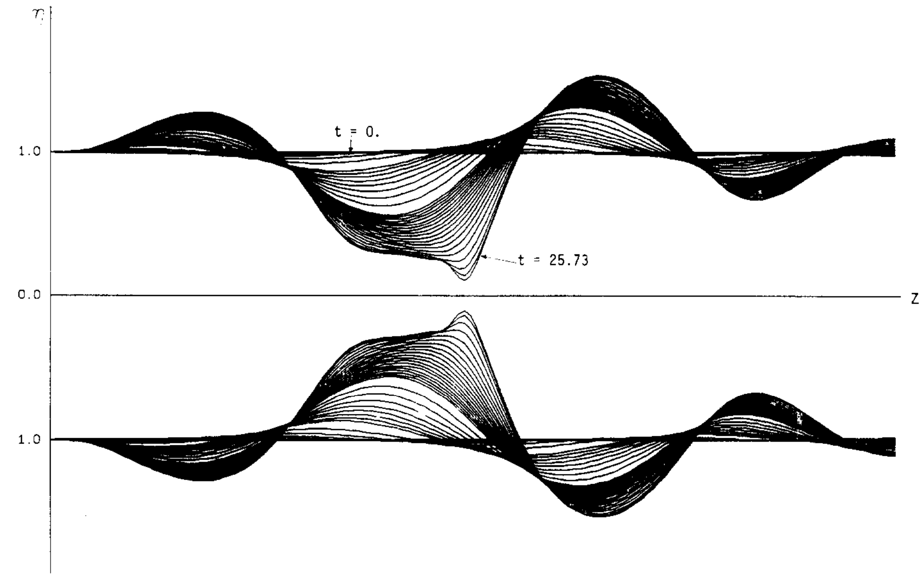

FIG. 2. Successive jet profiles.

To test the suitability of the end conditions, computations were carried out with 35 and 50 longitudinal grid points. The wavelength of the fastest growing disturbance (approx. 4.6 nozzle diameters) was estimated to be the same in both instances and in agreement with that obtained earlier for the infinite jet. This wavelength was also found to be insensitive to the initial conditions imposed. Furthermore, and *crucially*, there was no apparent distortion of surface waves as they passed out of the computation field. The internal velocity distribution of the jet was found to become nearly uniform a short distance from the nozzle orifice. These findings, though convincing to ourselves, must be regarded as tentative, since circumstances precluded truly extensive numerical explorations.

Figure 2 shows successive jet profiles for the physical situation hypothesized above. Both the incipient formation of a drop and of a satellite are to be seen.

We conclude that the proposed algorithms for Eulerian coordinates do permit the successful simulation of jet breakup induced by surface tension, and it may be anticipated that these algorithms have utility well beyond the present application. Furthermore, and as expected, our results support Rayleigh's assumption of an infinitely-long jet, with the consequent possibility of a Lagrangian treatment.

REFERENCES

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F. SHOKOOHI

Automotive Technologies International, Inc.
Boonton Township, New Jersey 07005

H. G. ELROD

Columbia University
New York, New York 10027